

15 Negative Numbers

15.1 Addition and Subtraction

In this section we consider how to add and subtract when working with negative numbers. This is done using a number line.



To add a positive number, move to the right on a number line.

To add a negative number, move to the left on a number line.



Example 1

Calculate:

(a) $-3 + 8$

(b) $4 + (-2)$

(c) $-8 + 5$

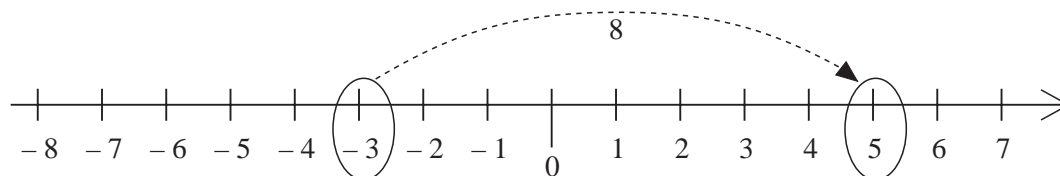
(d) $-1 + (-3)$



Solution

(a) $-3 + 8$

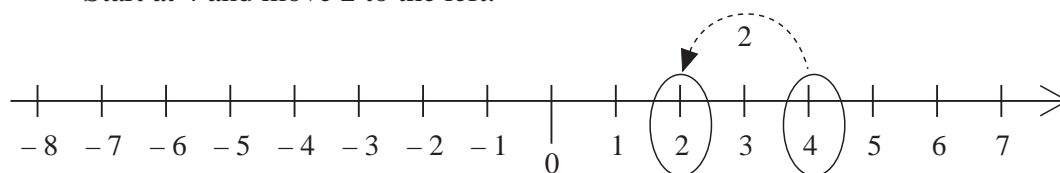
Start at -3 and move 8 to the right:



So, $-3 + 8 = 5$.

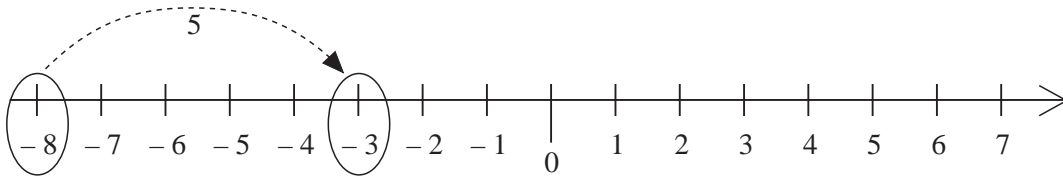
(b) $4 + (-2)$

Start at 4 and move 2 to the left:



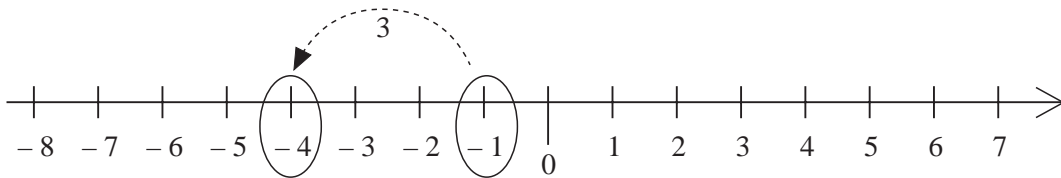
So, $4 + (-2) = 2$.

(c) $-8 + 5$

Start at -8 and move 5 to the right:

So, $-8 + 5 = -3$.

(d) $-1 + (-3)$

Start at -1 and move 3 to the left:

So, $-1 + (-3) = -4$.

Subtraction is the *inverse* (or opposite) of addition, so

To *subtract a positive number*, move to the *left* on a number line.

To *subtract a negative number*, move to the *right* on a number line.

**Example 2**

Calculate:

(a) $5 - 7$

(b) $-2 - 3$

(c) $4 - (-2)$

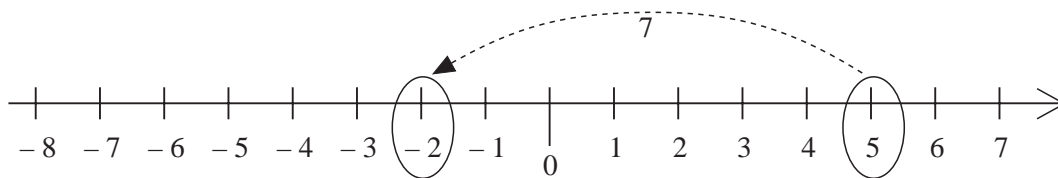
(d) $-5 - (-3)$



Solution

(a) $5 - 7$

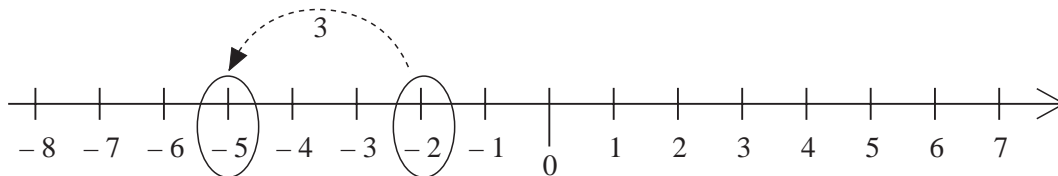
Start at 5 and move 7 to the left:



So, $5 - 7 = -2$.

(b) $-2 - 3$

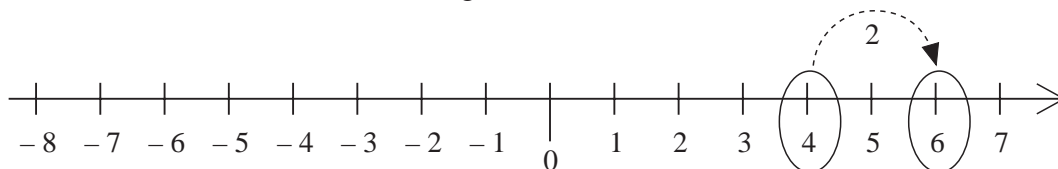
Start at -2 and move 3 to the left:



So, $-2 - 3 = -5$.

(c) $4 - (-2)$

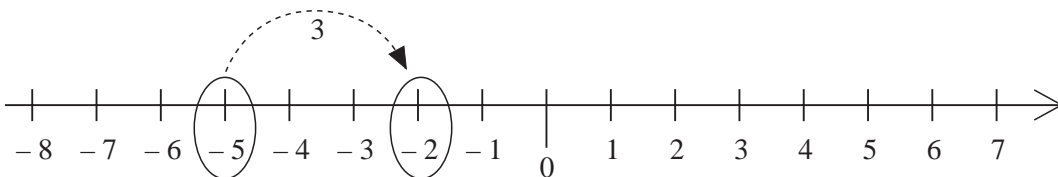
Start at 4 and move 2 to the right:



So, $4 - (-2) = 6$.

(d) $-5 - (-3)$

Start at -5 and move 3 to the right:



So, $-5 - (-3) = -2$.



Exercises

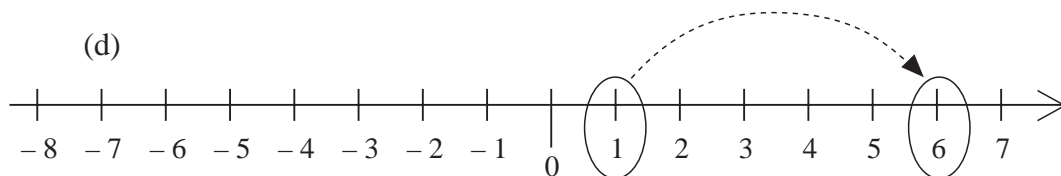
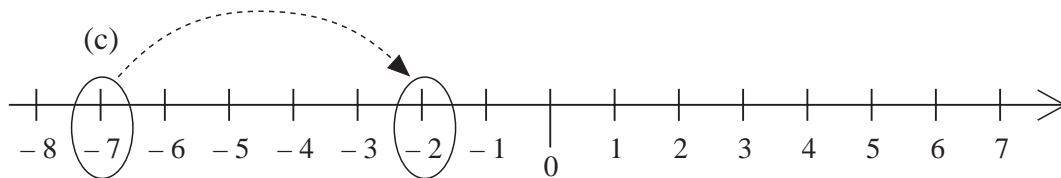
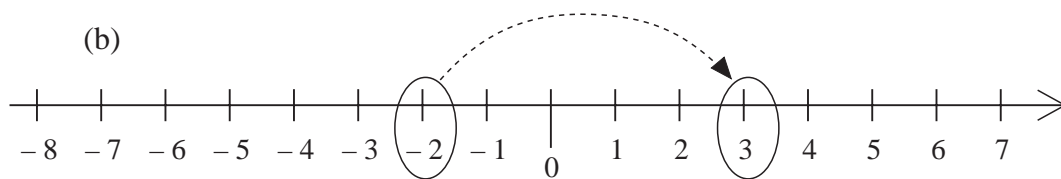
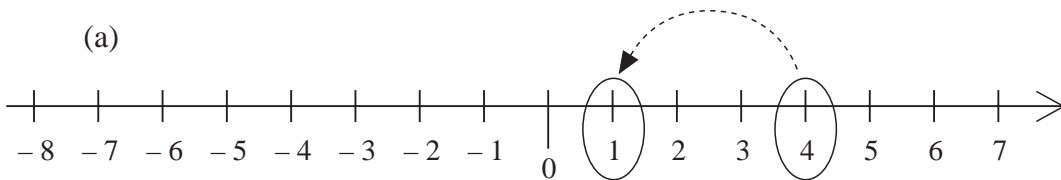
1. Use a number line to work out the following calculations:

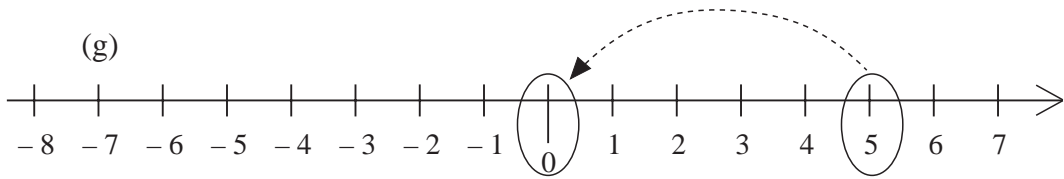
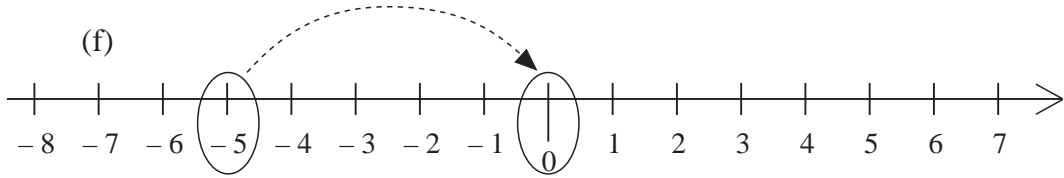
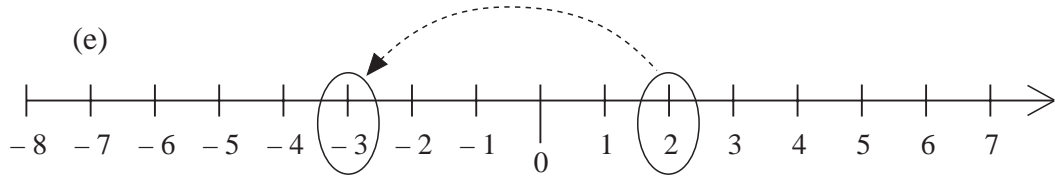
- | | | |
|-----------------|----------------|-----------------|
| (a) $-4 + 6$ | (b) $-5 + 8$ | (c) $-1 + 3$ |
| (d) $-4 + 7$ | (e) $2 + (-3)$ | (f) $-1 + (-4)$ |
| (g) $-2 + (-3)$ | (h) $-6 + 6$ | (i) $-7 + 4$ |
| (j) $-6 + 2$ | (k) $-7 + 2$ | (l) $5 + (-5)$ |

2. Use a number line to calculate:

- | | | |
|-----------------|----------------|-----------------|
| (a) $4 - 6$ | (b) $5 - 7$ | (c) $2 - 4$ |
| (d) $-1 - 1$ | (e) $-3 - 2$ | (f) $-4 - (-1)$ |
| (g) $3 - (-4)$ | (h) $5 - (-6)$ | (i) $8 - 12$ |
| (j) $-5 - (-1)$ | (k) $4 - 9$ | (l) $-4 - (-4)$ |

3. Write down the two possible sums that could be shown by each number line below:





4. Copy and complete this addition table:

+	-4	-2	0	2	4
-3					
-1					
1					
3					

5. Fill in the missing numbers on a copy of this addition table:

+		1		3	
-5	-5				-1
		-3			
				0	
	-2		0		

6. Copy these equations and fill in the missing numbers:
- (a) $4 + \dots = 1$ (b) $3 - \dots = -6$
 (c) $-6 + \dots = -7$ (d) $\dots + 7 = 2$
 (e) $\dots - 7 = -8$ (f) $4 - \dots = 7$
7. Write down the next 5 terms in each sequence:
- (a) 10, 8, 6, 4, ...
 (b) -10, -7, -4, -1, ...
 (c) 19, 14, 9, 4, ...
 (d) -1, -3, -5, -7, ...
 (e) -20, -16, -12, -8, ...
 (f) -5, -10, -15, -20 ...
 (g) 18, 15, 12, 9, ...
 (h) -16, -12, -8, -4, ...
8. Overnight, the temperature dropped from 5°C to -14°C . By how many degrees did the temperature fall?
9. One day the level of the water in a river was 30 cm *above* its average level. One week later it was 12 cm *below* its average level. How far did the water level drop in the week?
10. A chest of treasure was hidden in the year 64 BC and found in 284 AD. For how long was the chest hidden?

15.2 Multiplication and Division

In this section we look at how to multiply and divide negative numbers.



Example 1

- (a) Calculate

$$(-2) + (-2) + (-2) + (-2) + (-2).$$

- (b) Fill in the missing numbers:

$$(-2) + (-2) + (-2) + (-2) + (-2) = \dots \times (-2) = \dots$$

**Solution**

- (a) $(-2) + (-2) + (-2) + (-2) + (-2) = -10$
 (b) $(-2) + (-2) + (-2) + (-2) + (-2) = 5 \times (-2) = -10$

In Example 1 we see that

*a positive number multiplied by
 a negative number gives a negative answer.*

This table shows what happens to the sign of the answer when positive and negative numbers are *multiplied*:

×	+	-
+	+	-
-	-	+

The same table can be used for *division* of positive and negative numbers.

**Example 2**

Work out the following:

- (a) $5 \times (-7)$ (b) $(-8) \times (-10)$
 (c) $(-42) \div 6$ (d) $(-88) \div (-8)$

**Solution**

- (a) First calculate $5 \times 7 = 35$.

As a positive number is multiplied by a negative number, the answer will be negative:

$$5 \times (-7) = -35$$

- (b) First calculate $8 \times 10 = 80$.

Here a negative number is multiplied by a negative number, so the answer will be positive:

$$(-8) \times (-10) = 80$$

- (c) First calculate $42 \div 6 = 7$.

As a negative number is divided by a positive number, the answer will be negative:

$$(-42) \div 6 = -7$$

- (c) First calculate
- $88 \div 8 = 11$
- .

As a negative number is divided by a negative number, the answer will be positive:

$$(-88) \div (-8) = 11$$



Exercises

1. Calculate:

- (a) $(-7) \times 2$ (b) $(-4) \times 8$ (c) $(-2) \times (-5)$
 (d) $(-6) \times (-3)$ (e) $(-3) \times 7$ (f) $(-10) \times (-4)$
 (g) 8×4 (h) $3 \times (-6)$ (i) $(-7) \times (-2)$
 (j) $(-4) \times (-5)$ (k) $(-7) \times 0$ (l) $8 \times (-5)$

2. Calculate:

- (a) $(-10) \div (-2)$ (b) $(-15) \div 5$
 (c) $18 \div (-3)$ (d) $14 \div (-7)$
 (e) $(-21) \div (-3)$ (f) $(-45) \div 9$
 (g) $50 \div (-5)$ (h) $(-100) \div (-4)$
 (i) $80 \div (-2)$ (j) $26 \div (-13)$
 (k) $(-70) \div (-7)$ (l) $(-42) \div 7$

3. Copy and complete these multiplication tables:

(a)

\times	1	2	3	4
-1				
-2				
-3				
-4				

(b)

\times	1	0	-1	-2	-3
-4					
-2					
0					
1					

4. Copy and complete these multiplication tables:

(a)

×		-1	
2			
	-2	2	
-3			-9

(b)

×	-2		
	10		
-2		6	
3			-12

5. Copy these calculations, filling in the missing numbers:

(a) $\dots \times 5 = -20$

(b) $(-80) \div \dots = 4$

(c) $16 \times \dots = -32$

(d) $(-4) \times \dots = 32$

(e) $\dots \times (-3) = 12$

(f) $40 \div \dots = -8$

(g) $-8 \times \dots = 48$

(h) $-32 \div \dots = 4$

(i) $15 \times \dots = -60$

(j) $100 \div \dots = -25$

6. Write down the next 3 terms in each sequence:

(a) 1, -2, 4, -8, 16, ...

(b) -1, 2, -4, 8, -16, ...

(c) 1, -10, 100, -1000, ...

(d) 1, -3, 9, -27, ...

(e) -1, 5, -25, 125, ...

For each sequence, describe the rule that is used to calculate the next term.

7. Make 2 copies of this multiplication table and fill in the missing numbers in 2 different ways:

×				
	1			
		4		
			9	
				25

8. Calculate:

(a) $3 \times (-8) \times (-4)$

(b) $(-4) \times (-8) \times (-2)$

(c) $(-2) \times (-2) \times 2$

(d) $4 \times (-7) \times 2$

(e) $(-2) \times 8 \times (-4)$

(f) $(-6) \times (-2) \times (-1)$

9. Calculate:

(a) $\frac{(-3) \times (-4)}{(-2)}$

(b) $\frac{5 \times (-6)}{(-2)}$

(c) $\frac{(-7) \times (-5) \times (-2)}{5}$

(d) $\frac{8 \times (-9) \times 6}{(-2) \times (-3)}$

(e) $\frac{(-6) \times (-4)}{2}$

(f) $\frac{(-4) \times (-7) \times 3}{(-12)}$

10. Calculate:

(a) $(-6 + 10) \div (-2)$

(b) $(12 - 24) \div (-2)$

(c) $(6 + (-8)) \times (4 - 7)$

(d) $((-2) + 8) \times ((-4) + 2)$

(e) $((-4) \times 2) + (6 \times (-9))$

(f) $(8 \times (-2)) - ((-4) \times 8)$

11. Calculate:

(a) $(-6) \times (-3) + (-4)$

(b) $(-5) \times 4 - (-3)$

(c) $(-8) \times (-7) - 8 \times 7$

(d) $(-11) \times 4 + (-8) \times (-3)$